Constrained inefficiency and optimal taxation under uninsurable risks

Piero Gottardi[1] Atsushi Kajii[2] Tomoyuki Nakajima[2]

[1]European University Institute

[2]Kyoto University

.

Capital accumulation under incomplete markets

- Consider an economy with uninsurable background income risks.
  - there is no aggregate risk in the economy, but each individual faces income shocks, so markets are incomplete.
  - Thus typically, a competitive equilibrium is inefficient
  - Inefficiency is due to "incorrect prices" so there is a room for a welfare improving taxation which changes relative prices.
  - then which commodities should be taxed?
- Compare an equilibrium of this economy with an equilibrium in the complete market economy where the risks can be fully insured.
- Under precautionary saving (prudence), the amount of investment is larger in the incomplete markets than in the complete markets.
  - Idea: aggregate saving curve (i.e., supply curve) shifts to the "right" by precautionary motive.

Over Investment??

- Since a complete market equilibrium is efficient, one might think that this means:
  - in the incomplete markets equilibrium, investment is excessive.
  - reduction in investment by some policy improves welfare.
  - in particular, capital should be taxed to discourage investment.
- Although these have been said (and perhaps even accepted) in the literature,

1. we argue that both arguments are wrong,
2. we show how to think of these correctly.
3. present a formula which summarizes various welfare effects

Why wrong? (I)

- First, comparing the two economies makes little sense:
  - if the completed markets are available to begin with, there is no point in thinking of the incomplete markets.
  - Thus a presumption here must be that the market *cannot* be completed, i.e., there are only limited policy tools for welfare improvement.
- The question should be whether or not there is an welfare improving policy, and how it is characterized.
  - I.e., needs to define the set of allocations achievable by some policy, and ask which one is efficient among them - second best argument!
  - if we are to say "over-investment ", it should mean that there is a welfare improving policy which reduces investment.

Why wrong? (II)

- Secondly, an inefficient equilibrium *does not* mean that everybody is worse of than in an efficient equilibrium.
  - So moving from inefficient to efficient may be bad for many consumers - at least there are trade-offs
- Thus one must inevitably introduce some social welfare criterion for a policy evaluation.
- In particular, a social welfare function argument will favor income smoothing across households, and income smoothing and improving efficiency of capital use might not be the same thing.

How to think - the Purpose of this paper and Results

- We consider a simple two period model with background risks, and consider explicitly two kinds of policy tools:
- **The constrained (in)efficiency exercise**: direct control on capital and labor (but the prices are determined in free markets)
- **The optimal taxation exercise**: explicitly consider various tax - and - subsidy schemes, with a social welfare function
- Results: we obtain simple characterization results for welfare changes induced by those policy tools: they say welfare effects are decomposed in insurance effect and distribution effect
  - The formula indicates that the **heterogeneity of income distribution is important** to see if capital should be taxed or subsidized
    - Recall that the "wrong" conclusion was drawn independent of heterogeneity.

Literature

- Comparison between incomplete and complete markets economies:
  - Aiyagari (1994); Marcet et al (2007); etc.
- Constrained inefficiency of incomplete-markets equilibrium:
  - Geanakoplos and Polemarchakis (1986); Citanna et al (1998); **Davila et al (2005)**; etc.
    - Davila et al, despite a restrictive setup of inelastically supplied labor, addresses the constrained efficiency in this way.
    - In a companion paper, we deal with an infinite horizon version of this paper.
- Optimal taxation in incomplete-markets economies:
  - Aiyagari (1995); Conesa et al (2009); etc.

Plan of Talk

1. Model and definition of competitive equilibrium with uninsurable risks
2. analysis of constrained efficiency and optimality
3. analysis of optimal taxation
4. conclusion

Model (1/6): firm and consumers.

- Two periods, period 0 and 1, and a single consumption good (numéraire) in each period, which may be consumed or invested.
- Competitive markets with many consumers and one firm.
- Firm: produce consumption good in period 1, from labor (supplied in period 1) and capital (invested in period 0).
- I types of individuals, each type $i = 1, ..., I$ consists of a continuum of individuals of size 1.
- Each individual receives consumption good in period 0 and labor hours in period 1.
- The productivity of investment and that of labor are uncertain due to idiosyncratic shocks.
- But individuals of same type are ex ante identical and so their planned behavior is identical (symmetric) ex ante.

Model (2/6): uninsurable risks

- Source of idiosyncratic shock is described by a random variable θ_i, which works as follows:
  - if k_i units invested, $\rho_i^K(\theta_i)k_i$ units of the capital good is provided
  - labor hour $h_i^{\theta_i}$ will be chosen AFTER θ_i is observed, and $\rho_i^L(\theta_i)h_i^{\theta_i}$ **efficiency** units of labor is supplied.
  - Both ρ_i^L and ρ_i^K are non-decreasing and so higher θ_i means that the investment and the labor hours are more efficient (and valuable).
- So if the market return on capital is r and the wage is w, the second period income will be $wL_i^{\theta_i} + rK_i^{\theta_i}$ at θ_i where

$$L_i^{\theta_i} \equiv \rho_i^L(\theta_i)h_i^{\theta_i},\ K_i^{\theta_i} \equiv \rho_i^K(\theta_i)k_i.$$

- In particular, the income will be random but there is no market which provide insurance for the income.

Model (3/6): no aggregate uncertainty

- By assumption, no aggregate uncertainty; that is, the realization of θ_i's across the individuals of type i coincides with the population distribution with probability one.
  - For ease of exposition, I shall assume $\rho_i^K \equiv 1$, i.e., no shock for capital.
- Example: $\theta_i = 1.5$ or 0.5 with equal probability.
  - **With probability one**, $\frac{1}{2}$ of type i population sees 1.5, the other $\frac{1}{2}$ gets 0.5,
  - i.e., an individual knows half of the population will get a high θ_i and the other gets a low θ_i, but does not know which half he will belong to.
  - e.g., if type i individual plan to supply 3 units of labor hours when $\theta_i = 1.5$ and 1 unit when $\theta_i = 0.5$, then the aggregate per capita supply of the labor will be 2 for sure.
  - i.e., in aggregate, the demand and supply will only depend on the distribution of the population, not the individual fortunes.

Model (4/6): endowments and income

- A type i individual receives e_i units of the good in the first period, and $\bar{H}_i$ hours of labor time in the second.
- Each individual decides how much to invest k_i before learning his θ_i, but chooses labor hour $h_i^{\theta_i}$ after θ_i is learned.
- There are markets for the good as well as labor in **efficiency units** (wage w) and capital rental (return r). The price of the good is normalized to be one in every period.
- Note: the aggregate demand and supply are non-random, so the market clearing prices will not depend on the states.
- Writing c_i^0 for consumption in period 0, k_i for investment, $c_i^{\theta_i}$ for the consumption when θ_i is observed,
- period 0 budget is $c_i^0 + k_i = e_i$
- and period 1 budget is $c_i^{\theta_i} = wL_i^{\theta_i} + rK_i^{\theta_i}$ at every θ_i.

Model (5/6): preferences and consumer decision

- Preferences: with strictly concave vNM functions,

$$U_i = v_i(c_i^0) + \mathbf{E}\left[u_i\left(c_i^{\theta_i}, \bar{H}_i - h_i^{\theta_i}\right)\right]$$

- Taking into account the budget constraints, the first-order conditions characterize the individual choices:

$$v_i'(c_i^0) = \mathbf{E}\left[u_{ic}\left(c_i^{\theta_i}, \bar{H}_i - h_i^{\theta_i}\right) r\rho_i^K(\theta_i)\right]$$

$$u_{il}\left(c_i^{\theta_i}, \bar{H}_i - h_i^{\theta_i}\right) = u_{ic}\left(c_i^{\theta_i}, \bar{H}_i - h_i^{\theta_i}\right) w\rho_i^L(\theta_i), \quad \text{at each } \theta_i$$

- A firm with constant-returns-to-scale technology: per capita production function (CRS)

$$y = F(k, l)$$

- Define per-capita capital and labor by:

$$K \equiv \frac{1}{I}\sum_{i=1}^{I} K_i, \text{ and } L \equiv \frac{1}{I}\sum_{i=1}^{I} L_i$$

where by LLN assumption, for each type, the equals the mean, i.e.,

$$K_i \equiv \mathbf{E}(K_i^{\theta_i}) = k_i, \text{ and } L_i \equiv \mathbf{E}(L_i^{\theta_i})$$

Profit maximization condition:

$$r = F_K(K, L) \text{ and } w = F_L(K, L)$$

Competitive equilibrium: Definition

A competitive equilibrium is a collection $\left(w, r, \left(k_i, (h_i^{\theta_i} : \theta_i)\right)_{i=1}^{I}\right)$ such that

1. [utility max] for each i, $\left(k_i, (h_i^{\theta_i} : \theta_i)\right)$ maximizes utility given budget.
2. [profit max] r and w satisfy

$$r = F_K(K, L), \quad \text{and} \quad w = F_L(K, L)$$

   where:
3. [Market clearing for capital and labor] $K = \frac{1}{I} \sum k_i$, and $L = \frac{1}{I} \sum \mathbf{E} \left[\rho_i^L(\theta_i)\, h_i^{\theta_i}\right]$

- markets for the consumption good also clear because of budget constraints.

Standard Equilibrium

In an equilibrium $\left(w, r, \left(k_i, (h_i^{\theta_i} : \theta_i)\right)_{i=1}^{I}\right)$ labor hours $\left(h_i^{\theta_i}\right)_{\theta_i}$ can be seen as a function of the realization of θ_i, so are the other variables. Recall that a household is richer in high θ_i states:

A competitive equilibrium $\left(\hat{w}, \hat{r}, \left(\hat{k}_i, (\hat{h}_i^{\theta_i} : \theta_i)\right)_{i=1}^{I}\right)$ is said to be **standard** if for every i:
(1) $u_{ic}\left(\hat{c}_i^{\theta_i}, \hat{l}_i^{\theta_i}\right)$ is decreasing in θ_i where $\hat{c}_i^{\theta_i} = \hat{w}\hat{L}_i^{\theta_i} + \hat{r}\hat{K}_i^{\theta_i}$;
(2) both $\hat{K}_i^{\theta_i}$ and $\hat{L}_i^{\theta_i}$ are non-decreasing in θ_i.

- A standard equilibrium obtains under assumptions such as consumption and leisure are normal goods.

Constrained feasibility and efficiency

- Imagine a social planner who can control investments as well as labor choices, but not the other choices, especially the activity of the firm.
- That is, the prices are determined in the markets.
- A constrained feasible state: $\left(w, r, \left(k_i, \left\{h_i^{\theta_i} : \theta_i\right\}\right)_{i=1}^{I}\right)$, such that r and w satisfy

$$r = F_K(K, L), \quad \text{and} \quad w = F_L(K, L)$$

  where K and L are aggregate variables calculated from k_i and $h_i^{\theta_i}$
- Notice that the households' welfare is not only directly affected by the choice of k_i and $h_i^{\theta_i}$, but also indirectly affected through prices which change the income levels.

Over Investment

- Fix a standard equilibrium $\left(\hat{w}, \hat{r}, \left(\hat{k}_i, \left\{\hat{h}_i^{\theta_i} : \theta_i\right\}\right)_{i=1}^{I}\right)$ and we investigate welfare change locally around it.
- We say the equilibrium is **constrained efficient** if there is no way to improve every households' ex ante welfare.
- We say the equilibrium is **constrained optimal** if there is no way to improve a given weighted sum of households' ex ante utility functions.
  - I.e., we regard $\sum_{i=1}^{I} \lambda_i U_i$ as a social welfare function.
  - We will see that a standard equilibrium is likely to be constrained efficient in heterogenous economies, so we study optimality issue.
- We say there is **over investment** if a reduction of investment (holding labor constant) improves the welfare.
  - So this is a standard second best argument - whereas the literature only compared two equilibria in two different economies.

Local welfare change

- Suppose that the planner changes $(k_i)_{i=1}^{I}$ from the equilibrium values by infinitesimal amounts, (holding $h_i^{\theta_i}$ fixed) and hence the aggregate K (whereas L is kept fixed). The market prices adjust so that the economy is in a constrained feasible state.
- How much the utility of household i change?
  - "direct effect from changing k_i" $= 0$, because of the envelope property holds at the equilibrium
  - "indirect effect through changing r and w" - this will arise. In this sense, the change is due to Pecuniary externality effect.

- In conclusion, the local welfare change is obtained taking $\hat{k}_i$ and $\hat{h}_i^{\theta_i}$ in the utility function as if they are fixed:
- we have:

$$\left.\frac{\partial U_i}{\partial K}\right|_{(\hat{K},\hat{L})} = \left.\frac{\partial}{\partial K}\mathbf{E}\left[u_i\left(w\left(K,L\right)\hat{L}_i^{\theta_i} + r\left(K,L\right)\hat{K}_i^{\theta_i}, \bar{H}_i - \hat{h}_i^{\theta_i}\right)\right]\right|_{(\hat{K},\hat{L})}$$
$$= \mathbf{E}\left[u_{ic}\cdot\left.\frac{\partial}{\partial K}\left(w\left(K,L\right)\hat{L}_i^{\theta_i} + r\left(K,L\right)\hat{K}_i^{\theta_i}\right)\right|_{(\hat{K},\hat{L})}\right]$$
$$= \mathbf{E}\left[u_{ic}\cdot\left(\frac{\partial w}{\partial K}\cdot\hat{L}_i^{\theta_i} + \frac{\partial r}{\partial K}\cdot\hat{K}_i^{\theta_i}\right)\right], \quad (1)$$

- From the Euler equation, $F_K(K,L)K + F_L(K,L)L = F(K,L)$, we have:

$$\frac{\partial r}{\partial K} \cdot K + \frac{\partial w}{\partial K} \cdot L = 0. \tag{2}$$

and so

$$\left.\frac{\partial U_i}{\partial K}\right|_{(\hat{K},\hat{L})} = \left\{\mathbf{E}\left[u_{ic} \cdot \left(\hat{K}_i^{\theta_i} - \hat{K}_i\right)\right] + \mathbf{E}\left[u_{ic}\right]\left(\hat{K}_i - \hat{K}\right)\right\}\frac{\partial r}{\partial K} \tag{3}$$

$$+ \left\{\mathbf{E}\left[u_{ic} \cdot \left(\hat{L}_i^{\theta_i} - \hat{L}_i\right)\right] + \mathbf{E}\left[u_{ic}\right]\left(\hat{L}_i - \hat{L}\right)\right\}\frac{\partial w}{\partial K},$$

- Refer to $\mathbf{E}\left[u_{ic} \cdot \left(L_i^{\theta_i} - \hat{L}_i\right)\right]$ as the **insurance effect**, and $\mathbf{E}\left[u_{ic}\right]\left(\hat{K}_i - \hat{K}\right)$ and $\mathbf{E}\left[u_{ic}\right]\left(\hat{L}_i - \hat{L}\right)$ as the **distribution effects**.

Insurance effect

- $\mathbf{E}\left[u_{ic} \cdot \left(\hat{L}_i^{\theta_i} - \hat{L}_i\right)\right] < 0$ at the standard equilibrium.
  - as $\theta_i \uparrow$, income $\uparrow$ so $u_{ic} \downarrow$, and $\hat{L}_i^{\theta_i} \uparrow$
- This measures the welfare (loss) due to randomness **within** a type.
  - $\mathbf{E}\left[u_{ic} \cdot \left(\hat{K}_i^{\theta_i} - \hat{K}_i\right)\right] = 0$ for capital, since there is no shock on capital.
- Since $\frac{\partial w}{\partial K} = F_{KL} > 0$, as far as the insurance effect is concerned, $\left.\frac{\partial U_i}{\partial K}\right|_{(\hat{K},\hat{L})}$ tends to be negative (i.e., reduction of investment is welfare improving.

Distribution effects

- The signs of $\mathbf{E}[u_{ic}]\left(\hat{K}_i - \hat{K}\right)$ and $\mathbf{E}[u_{ic}]\left(\hat{L}_i - \hat{L}\right)$ depend on the relative position of type i in the whole economy.
- Then the distribution effects can get large if the heterogeneity among the types is large.
- Say that the equilibrium is monotonic if $\hat{K}_i$ and $\hat{L}_i$ are negatively correlated.
  - That is, those who have a higher capital income (capitalists) tend to have lower than average labor income, and those who have a higher labor income (labor) tend to have lower than average capital income.
- Then the distribution effects is large when income discrepancy between capitalists and labor is large.

Over-saving in homogeneous economy

- But the distribution effects will be small when the agents are homogeneous enough.
- At the extreme, they are both zero if all the households are ex ante identical.
- We have seen that the welfare effect from the insurance effect is negative.
- So we conclude that **there is over investment** at a standard equilibrium in economies **with homogeneous enough households**.

Heterogeneity matters.

- But as the heterogeneity (i.e., difference in wealth) increases, the distribution effect will get larger.
- On the other hand, the insurance effect is more to do individual risk bearing, so it won't change much.
- Thus one can expect that if the heterogeneity is large, those with low e_i would rather want K increase.
- Intuitive - if my primary source of income is from labor, I would rather see that the return on capital r decreases, which makes the relative price of labor higher.

The model with taxes

- With taxes, the basic idea remains the same - we do a second best argument
- We first need to define "feasible" allocations, which are competitive equilibria with tax - subsidy
- Basic problem for consumers: a consumer maximizes utility given taxes and lump-sum subsidy subject to budget constraints for type-i individual with taxes:

$$
\begin{aligned}
c_i^0 &= e_i - k_i \\
c_i^{\theta_i} &= (1-\tau_K)rK_i^{\theta_i} + (1-\tau_L)wL_i^{\theta_i} + T_i^{\theta_i}
\end{aligned}
$$

where

- τ_K and τ_L are the tax rates on capital and labor,
- $T_i^{\theta_i}$ are lump-sum transfers/taxes.

Equilibrium with a tax-subsidy

- **An equilibrium with a tax-subsidy** is a collection $\left(w, r, \left(\tau_K, \tau_L, (T_i^{\theta_i} : \theta_i)_{i=1}^I\right), \left(k_i, (h_i^{\theta_i} : \theta_i)\right)_{i=1}^I\right)$ such that

1. for each i, $\left(k_i, (h_i^{\theta_i} : \theta_i)\right)$ solves the type-i utility max problem;
2. prices satisfy

$$r = F_K(K, L), \quad \text{and} \quad w = F_L(K, L)$$

   where $K = \frac{1}{I} \sum_i E_{\theta_i} \rho_i^K(\theta_i) k_i$ and $L = \frac{1}{I} \sum_i E_{\theta_i} \rho_i^L(\theta_i) h_i^{\theta_i}$;

3. the planner's balance condition + restriction on transfer:

$$T_i^{\theta_i} = \tau_L wL + \tau_K rK$$

   - I.e., every individual gets the same amount of transfer independent of personal state, which must be financed by taxes.
     - In the paper, more variations are considered

Observations (1)

- The domain for the second best arguments = the consumption allocations which arise in some tax-subsidy equilibria.
- There is a tax subsidy equilibrium with $\tau_K = \tau_L = 0$, which is identical to a (standard) competitive equilibrium we are focusing on;
    - set $T_i^{\theta_i} \equiv 0$, then the planner's balance condition is trivially satisfied. At a competitive equilibrium conditions 1 and 2 are satisfied.
    - So a competitive equilibrium is constrained feasible.
- The system of equations defining the tax-subsidy equilibrium have two extra variables: we can take τ_K and τ_L as if they are parameters and discuss the corresponding tax-subsidy equilibrium.
    - i.e., equilibrium variables including the subsidy can be regarded as functions of (τ_K, τ_L), locally around a fixed (standard) equilibrium.

Observations (2)

- Fix a standard equilibrium.
- Intuitively, other things being equal, a tax should discourage the use of the respective production factor.
- But do not forget general equilibrium effects - tax comes back to somebody as income, for instance.
- So the tax-subsidy scheme should play two extra roles:
- It will generate income transfers across different types
- It will also serve as an insurance device - notice that one gets a fixed amount of subsidy irrespective of the shock, which is nothing but an insurance.

Utility around the equilibrium

- Want to see the following utility level at tax equilibria around the fixed standard equilibrium

$$U_i\left(\tau_K,\tau_L\right) := v_i(e_i - k_i) + \mathbf{E}\left[u_i\left((1-\tau_K)rK_i^{\theta_i} + (1-\tau_L)wL_i^{\theta_i} + T_i^{\theta_i}, \bar{H}_i - h_i^{\theta_i}\right)\right]$$

  where the endogenous tax equil. var's are functions of $\left(\tau_K,\tau_L\right)$.
- So differentiate $U_i\left(\tau_K,\tau_L\right)$ w.r.t. τ_K and τ_L, and evaluate them at $\left(\tau_K,\tau_L\right) = 0$
- **KEY POINT**: the effects via the choices of households (i.e., k_i, $h_i^{\theta_i}$) *at the equilibrium* will be zero by the envelope property.
- Thus we only need to look at the changes via changes in prices (and direct tax-subsidy terms).

Derivatives:

- So we need to differentiate, $\mathbf{E}\left[u_i\left((1-\tau_K)rK_i^{\theta_i}+(1-\tau_L)wL_i^{\theta_i}+T_i^{\theta_i}, \bar{H}_i-h_i^{\theta_i}\right)\right]$, taking $h_i^{\theta_i}$, $L_i^{\theta_i}$ and $K_i^{\theta_i}$ as if they are fixed: we get

$$\left.\frac{\partial \mathcal{U}_i}{\partial \tau_K}\right|_{\tau=0} = \mathbf{E}\left[u_{ic}\cdot\left(\hat{K}_i^{\theta_i}\left(\frac{\partial r}{\partial \tau_K}-\hat{r}\right)+\hat{L}_i^{\theta_i}\frac{\partial w}{\partial \tau_K}\right)\right],$$
$$\left.\frac{\partial \mathcal{U}_i}{\partial \tau_L}\right|_{\tau=0} = \mathbf{E}\left[u_{ic}\cdot\left(\hat{K}_i^{\theta_i}\frac{\partial r}{\partial \tau_L}+\hat{L}_i^{\theta_i}\left(\frac{\partial w}{\partial \tau_L}-\hat{w}\right)\right)\right]$$

where the partial derivatives are evaluated at the equilibrium.

Signs

- To determine the sign of these, assume that the prices changes in natural directions:

$$\left.\frac{\partial r(\tau_K, \tau_L)}{\partial \tau_K}\right|_{\tau=0} > 0, \quad \left.\frac{\partial}{\partial \tau_K}\left[(1-\tau_K) r(\tau_K, \tau_L)\right]\right|_{\tau=0} < 0,$$
$$\left.\frac{\partial w(\tau_K, \tau_L)}{\partial \tau_L}\right|_{\tau=0} > 0, \quad \left.\frac{\partial}{\partial \tau_L}\left[(1-\tau_L) w(\tau_K, \tau_L)\right]\right|_{\tau=0} < 0$$

- i.e., the before - tax price increases, while the net after-tax price decreases.
- Since F_K and F_L are homogeneous of degree zero, differentiating the profit maximization condition (which holds at *any* tax equilibrium), for $a = K, L$,

$$\hat{K}\frac{\partial r}{\partial \tau_a} + \hat{L}\frac{\partial w}{\partial \tau_a} = 0.$$

  - Thus in particular, $\frac{\partial w}{\partial \tau_K} < 0$ and $\frac{\partial r}{\partial \tau_L} < 0$.

Change in Utility: effect of capital taxation

So by subtracting this expression, we can re-write the derivative as follows:

$$
\begin{aligned}
\left.\frac{\partial \mathcal{U}_i}{\partial \tau_K}\right|_{\tau=0} = &\underbrace{\mathbf{E}\left[u_{ic} \cdot (\hat{K}_i^{\theta_i} - \hat{K}_i)\right]}_{0} \underbrace{\left(\frac{\partial r}{\partial \tau_K} - \hat{r}\right)}_{-} \qquad \text{(insurance: } 0\text{)} \\
&+ \underbrace{\mathbf{E}\left[u_{ic} \cdot (\hat{K}_i - \hat{K})\right]}_{+-} \underbrace{\left(\frac{\partial r}{\partial \tau_K} - \hat{r}\right)}_{-} \qquad \text{(distribution: } +-\text{)} \\
&+ \underbrace{\mathbf{E}\left[u_{ic} \cdot (\hat{L}_i^{\theta_i} - \hat{L}_i)\right]}_{-} \underbrace{\frac{\partial w}{\partial \tau_K}}_{-} \qquad \text{(insurance: } +\text{)} \\
&+ \underbrace{\mathbf{E}\left[u_{ic} \cdot (\hat{L}_i - \hat{L})\right]}_{+-} \underbrace{\frac{\partial w}{\partial \tau_K}}_{-} \qquad \text{(distribution: } +-\text{)}
\end{aligned}
$$

Insurance and Distribution Effects

- Notice that we get the same insurance effect and the distribution effects in the expression.
- Roughly speaking, we will get conclusions as follows: tax the factor of production which is over used in the equilibrium.
- However, whether or not a factor is over used depends on the heterogeneity of the economy.
- Indeed, the signs depend on the relative position of type i in the economy: Roughly speaking, if the heterogeneity is only due to the initial wealth,

	"capitalist"	"labor"
$\mathbf{E}\left[u_{ic}\right](\hat{K}_i - \hat{K})\left(\frac{\partial r}{\partial \tau_K} - \hat{r}\right)$	-	$+$
$\mathbf{E}\left[u_{ic}\right](\hat{L}_i - \hat{L})\frac{\partial w}{\partial \tau_K}$	$+$	-

- Hence the households' interests tend to disagree: some prefers a negative tax on capital.

An Inefficiency result

- *If the economy is homogeneous enough*, then a (standard) competitive equilibrium is constrained INEFFICIENT in the sense that there is a tax equilibrium which makes every household better off ex ante. Moreover, a **positive capital** tax rate improves everybody's welfare.
- Remarks:
  1. Note that this argument DOES NOT rely on whether or not a complete market equilibrium has a lower saving than the (standard) equilibrium - i.e., the argument does not rely on the prudence of households (at least not directly)
  2. It is also not hard to construct examples where an equilibrium exhibit under investment and then some households better off by a **negative** capital tax
  3. Similar formula holds for labor taxation

Social Welfare and welfare change

- To go further we need to evaluate trade-offs of households' gains and losses. So consider the welfare function:

$$\mathcal{W}(\tau_K, \tau_L) \equiv \sum_{i=1}^{I} \lambda_i U_i(\tau_K, \tau_L)$$

where λ_i are (fixed) the Pareto weights on type-i individuals.

- We want to find out the derivatives of $\mathcal{W}$ with respect to (τ_K, τ_L)
- If $\frac{\partial}{\partial \tau_K}\mathcal{W} > 0$, then (small) tax on capital is welfare improving.
- similarly, if $\frac{\partial}{\partial \tau_L}\mathcal{W} > 0$, then (small) tax on labor is welfare improving

Welfare Change: effect of capital taxation

$$\left.\frac{\partial \mathcal{W}}{\partial \tau_K}\right|_{\tau=0} = \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_i \cdot (K_i^{\theta_i} - K_i)\right]}_{0} \underbrace{\left(\frac{\partial r}{\partial \tau_K} - r\right)}_{-} \qquad \text{(insurance: } 0\text{)}$$

$$+ \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_i \cdot (K_i - K)\right]}_{-} \underbrace{\left(\frac{\partial r}{\partial \tau_K} - r\right)}_{-} \qquad \text{(distribution: } +\text{)}$$

$$+ \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_i \cdot (L_i^{\theta_i} - L_i)\right]}_{-} \underbrace{\frac{\partial w}{\partial \tau_K}}_{-} \qquad \text{(insurance: } +\text{)}$$

$$+ \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_i \cdot (L_i - L)\right]}_{+} \underbrace{\frac{\partial w}{\partial \tau_K}}_{-} \qquad \text{(distribution: } -\text{)}$$

Welfare Change: effect of capital taxation - idea

- So roughly speaking, the capital should be taxed unless the distribution effect on labor is very large. (NB. taxing capital tends to reduce the wage which is bad news for labor income)
  - if the economy is homogeneous enough, $\frac{\partial}{\partial \tau_K}\mathcal{W} > 0$ must hold by the inefficiency result.)
- Why should we expect $\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_{ic} \cdot (\hat{K}_i - \hat{K})\right] < 0$?
- u_{ic} is negatively correlated with income $\rightarrow$ $COV(u_{ic}, \hat{K}_i - \hat{K}) < 0$ if types with $\hat{K}_i > \hat{K}$ tend to be rich.
- Why should we expect $\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_{ic} \cdot (\hat{L}_i - \hat{L})\right] > 0$?
- type with $L_i > L$ is relatively poor (their K_i is relative small)

Welfare Change: numerical example

- Obviously if the economy is homogeneous enough, $\frac{\partial}{\partial \tau_K}\mathcal{W} > 0$ must hold by the inefficiency result.
- But an example can easily constructed where $\frac{\partial}{\partial \tau_K}\mathcal{W} < 0$:
  - Two types, the initial endowment, $(e_i)_{i=1}^{I}$, is the only difference: same additively separable CRRA utility function (with parameter values Macro people like); same labor endowment; same distribution of θ_i, Production:
$$F(K, L) = K^{0.36}L^{1-0.36}$$

Welfare Change: no lump sum transfer case

- Recall that the insurance effect says a positive tax is welfare improving for everybody.

- So unless the distribution effect is too large, the planner should always be interested in
- taxing labor for insurance purpose, and also
- taxing on capital since it might reduce the income of the rich and subsidize the poor.

- But then what happens if direct subsidy is not allowed: i.e., we impose further that $T_i^{\theta_i} = \tau_K K + \tau_L L = 0$, which would imply $\frac{d\tau_L}{d\tau_K} = -\frac{rK}{wL}$?

Welfare Change: no direct subsidy

$$
\begin{aligned}
\left.\frac{d\mathcal{W}}{d\tau_K}\right|_{\tau=0} &= \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_{ic}\cdot\left(K_i^{\theta_i}-K_i\right)\right]}_{0}\underbrace{\left(\frac{dr}{d\tau_K}-r\right)}_{-} \\
&+ \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_{ic}\cdot\left(K_i-K\right)\right]}_{-}\underbrace{\left(\frac{dr}{d\tau_K}-r\right)}_{-} \\
&+ \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_{ic}\cdot\left(L_i^{\theta_i}-L_i\right)\right]}_{-}\underbrace{\left(\frac{dw}{d\tau_K}-w\frac{d\tau_L}{d\tau_K}\right)}_{+} \\
&+ \underbrace{\sum_i \lambda_i \mathbf{E}_{\theta_i}\left[u_{ic}\cdot\left(L_i-L\right)\right]}_{+}\underbrace{\left(\frac{dw}{d\tau_K}-w\frac{d\tau_L}{d\tau_K}\right)}_{+}
\end{aligned}
$$

Should we tax capital when there is "over saving"?

- Because a positive tax on capital must imply subsidizing labor, the positive insurance effect from labor affects negatively when capital is taxed.
- Thus if the insurance effect from labor is large, then it is appropriate to **subsidize** capital to take advantage of the insurance effect.
- That is, when the economy is homogeneous enough (i.e., the insurance effect is more important), capital should be subsidized, rather than taxed as in the previous case.
    - it turns out, negative capital tax is desirable exactly when there is over investment.
- Of course they are not contradicting to each other - the sign of optimal tax depends on the set of available policy tool.

Conclusion: right intuition

- You might think, if there is too much capital, why on earth subsidizing capital can ever be desirable?

1. **Too much in what sense**?: The set of feasible consumption allocations supported in tax equilibrium does not contain the complete market equilibrium allocations.
   - Indeed, we have seen if the distribution effect is more important, capital should be *increased*.
2. Tax discourages activity - correct. But the tax revenue offers direct income transfers which is good for the economy assuming the welfare function.

- **Lesson: whether or not something is excessive or not must be judged within a set of feasible policy tools.**
  - Even if capital is deemed excessive within some set, it does not necessarily mean that capital should be taxed!